

# Probing the density dependence of symmetry energy via multifragmentation at sub-saturation densities

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Symmetry energy for asymmetric nuclear matter at sub-saturation densities was investigated in the framework of an isospin-dependent quantum molecular dynamics model. Single ratio of neutrons and protons is compared, for the first time, with the experimental data of Famiano *et al.* We have also performed the comparison for double ratio with experimental as well as different theoretical results of BUU97, IBUU04, BNV and ImQMD models. It is found that the double ratio predicts the softness of symmetry energy, which is little underestimated in single ratio. Further, the study of single ratio is extended for different kind of fragments, while, double ratio is for different neutron-rich isotopes of Sn.

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## I. INTRODUCTION

One of the most important challenges in heavy-ion physics is the determination of the isospin dependence of nuclear equation of state (NEOS), which plays very important role at low energy phenomena like nuclear structure, nuclear astrophysics [1], fusion, cluster radioactivity [2] etc; intermediate energy phenomena like multifragmentation, stopping, flow [3–26] etc.; and at last high energy phenomena like pion and kaon production [27, 28] etc. The symmetry energy is found to be the prominent candidate to study the isospin dependence of NEOS. In the past years, many studies are performed on the density dependence of symmetry energy at sub-saturation densities by using isotopic scaling [7–11], isobaric ratio [12], single and double ratios [13–17, 19, 20], isospin diffusion [6, 21], isospin distillation/fractionation [6, 22], and isospin migration/drift [4, 6, 23] etc. Apart from these, transverse and elliptic flow of neutrons and protons are also considered as a good candidate to emphasize on the importance of density dependence of symmetry energy [24–26]. Even with the help of these studies, the exact determination of symmetry energy is still under the way.

In present work, we only want to address the effect of symmetry energy on kinetic energy spectra of nucleons as well as the neutrons to protons ratio parameters. The later one was considered to be the ever first prominent candidate to extract the density dependence of symmetry energy.

Before we discuss the contents of the present work, let us have some highlights of the single and double ratio in heavy ion collisions. The single ratio study in heavy-ion collisions have already been done by different experimental and theoretical groups [13, 14, 20]. In the experiments, near Fermi energy, Hilscher *et al.* [14] found

that single ratio of pre-equilibrium nucleons is consistently higher than that of projectile-target system and it can not be explained by the Coulomb effects alone. Another experimental observation is the ratio of free neutrons and protons from two isotopic systems at 26 MeV/nucleon. A lot of interesting observations are made from the data. Schroder *et al.* [29] also systematically studied the spectra of pre-equilibrium neutrons and protons in both isospin symmetric and asymmetric systems. Recently, at NSCL/MSU Famiano *et al.* [13] measured the single and double ratios of free neutrons to protons for  $^{112}\text{Sn} + ^{112}\text{Sn}$  and  $^{124}\text{Sn} + ^{124}\text{Sn}$  at 50 MeV/nucleon. The results of double ratio of the above data have also been reproduced by different theoretical models, such as BUU97 [15], IBUU04 [16], BNV [17] and ImQMD [19]. Even, there are a lot of uncertainties in the determination of symmetry energy in term of different parameters like cross-section, symmetry energy coefficient, impact parameter and method of clusterization etc.

However, no study exists in the literature where the comparison of single ratio of neutrons to protons is performed with the experimental data. One step ahead, the single ratio for the fragments is still poorly known in the literature. A few studies existed from the BNV and IBUU04 calculations for the single ratio using the intermediate mass fragments (IMF's), which is only limited for the small range of the kinetic energy [6, 18], however, isospin distillation/fractionation is studied up to higher kinetic energy by Li *et al.* [30]. In extension of single ratio to double ratio, no one has tried to compare the double ratio findings for experiments and theories at one place to see which one is the most appropriate model and symmetry energy form. It is also absent from the literature that what is the affect on the double ratio if we consider the series of isotopes with different isospin contents? From all these gaps, it seems interesting to perform study on the single and double ratios simultaneously.

In this paper, we focus on the comparative study of single and double ratios of neutrons to protons with the

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experimental data of the MSU/NSCL collaborations [13]. Moreover, in addition to the comparison with the experimental data, our results of IQMD model (initially developed by Hartnack *et al.* [31]) are also compared with other studies of BUU97, IBUU04, BNV and ImQMD models. The study of single ratio is extended for different kinds of fragments up to higher kinetic energy, while, the double ratio is investigated for different neutron-rich systems having different isospin content.

The article is organized as follow: we discuss the model briefly in Sec. II. Our results and discussions are given in Sec. III and we summarize the results in Sec. IV.

## II. FORMALISM: ISOSPIN DEPENDENT QUANTUM MOLECULAR DYNAMICS MODEL (IQMD)

In IQMD model [5, 26, 31], nucleons are represented by the wave packets, just like the QMD model [32]. These wave packets of the target and projectile interact by the full Skyrme potential energy, which is represented by  $U$  and is given as:

$$U = U_\rho + U_{Coul}. \quad (1)$$

Here  $U_{Coul}$  is the Coulomb energy, and  $U_\rho$  is originated from the density dependence of the nucleon optical potential and is given as:

$$U_\rho = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta}{\gamma + 1} \frac{\rho^{\gamma+1}}{\rho_0^\gamma} + \frac{C_{s,p}}{2} \left( \frac{\rho}{\rho_0} \right)^{\gamma_i} \delta^2 \rho, \quad (2)$$

where  $\delta = \left( \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \right)$ ;  $\rho = \rho_n + \rho_p$ ,  $\rho_n$  and  $\rho_p$  are the neutron and proton densities, respectively. The densities  $\rho$ ,  $\rho_n$  and  $\rho_p$  has the dimensions of  $fm^{-3}$ .

First two of the three parameters of the Eq. 2 ( $\alpha$  and  $\beta$ ) are determined by demanding that at normal nuclear matter densities, the binding energy should be equal to 16 MeV and the total energy should have minimum at  $\rho_0$ . The third parameter  $\gamma$  is usually treated as a free parameter. Its value is given in term of the compressibility:

$$\kappa = 9\rho^2 \frac{\partial^2}{\partial \rho^2} \left( \frac{E}{A} \right). \quad (3)$$

The different values of compressibility give rise to a soft and a hard equation of state. The soft equation of state is employed in the present study with the parameters  $\alpha = -356$  MeV,  $\beta = 303$  MeV and  $\gamma = 7/6$  corresponding to isoscalar compressibility of  $\kappa = 200$  MeV. In the calculations, we use the isospin dependent in-medium cross section in the collision term and the Pauli blocking effects just like QMD model [32]. The third term in the Eq. 2 is the symmetry potential energy for a finite nuclear matter. The symmetry energy per nucleon employed in the simulation is the sum of the kinetic and potential term.

So, the total symmetry energy is given as:

$$E_{Sym}(\rho) = \frac{C_{s,k}}{2} \left( \frac{\rho}{\rho_0} \right)^{2/3} + \frac{C_{s,p}}{2} \left( \frac{\rho}{\rho_0} \right)^{\gamma_i}, \quad (4)$$

where,  $C_{s,k} = 25$  MeV from the Fermi Dirac distribution, which is well explained in Ref. [33], is known as symmetry kinetic energy coefficient, while,  $C_{s,p} = 35.19$  MeV is parametrized on the basis of the experimental value of the symmetry energy, is known as symmetry potential energy coefficient. On the basis of  $\gamma_i$  value, symmetry energy is divided into two types with  $\gamma_i = 0.5$  and  $\gamma_i = 1.5$ , corresponds to the soft and stiff symmetry energies, respectively. Note that the  $\gamma$  used in the determination of equation of state and  $\gamma_i$  used in the determination of symmetry energy are the different parameters. The interesting feature of symmetry energy is that, its value increases with decreasing  $\gamma_i$  at sub-saturation densities, while, opposite is true at supra-saturation densities. In other words, the soft symmetry energy is more pronounced at sub-saturation densities, while the stiff symmetry energy at supra-saturation densities.

The cluster yields are calculated by means of the coalescence model, in which particles with relative momentum smaller than  $P_{Fermi}$  and relative distance smaller than  $R_0$  are coalesced into a cluster. The value of  $R_0$  and  $P_{Fermi}$  for the present work are 3.5 fm and 268 MeV/c, respectively.

## III. RESULTS AND DISCUSSION

In the present study, we simulate thousands of events for the isotopes of Sn, namely  $^{112}\text{Sn} + ^{112}\text{Sn}$ ,  $^{124}\text{Sn} + ^{124}\text{Sn}$  and  $^{132}\text{Sn} + ^{132}\text{Sn}$  at incident energy of 50 MeV/nucleon by using the soft and stiff symmetry energy having  $\gamma_i = 0.5$  and 1.5, respectively. The collision geometry for the study is from semi-central to semi-peripheral one by keeping in mind the importance of impact parameter of NSCL/ MSU collaboration's experimental results. As discussed earlier, the soft equation of state with an isospin dependent NN cross sections of  $\sigma_{med} = \left( 1 - 0.2 \frac{\rho}{\rho_0} \right) \sigma_{free}$  is employed. The single and double ratio is considered as a point of importance in the present study. The neutrons to protons ratio is among the first observables that was proposed a possible sensitive probe for symmetry energy prediction [13, 14, 20]. This ratio is studied for the free nucleons, light charged particles (LCP's) (having charge number of 1 and 2) and intermediate mass fragments (IMF's) (having charge between 3 and  $Z_{tot}/6$ ), where  $Z_{tot}$  is the total charge of the projectile and target under study. The single ratio is just the ratio of neutrons to protons and is represented in the study by the  $R_{N/Z}$ , while double ratio is the ratio of the single ratios of any two isotopes of the Sn. In order to study the systematics of the isospin effect, the single ratio of the isotope with more number of neutrons is always

mentioned in the numerator, when double ratio is calculated. Mathematically, the double ratio is represented by  $DR_{N/Z}$  and is given as :

$$DR_{N/Z} = \frac{R_{N/Z}^{neutron\ rich}}{R_{N/Z}^{neutron\ weak}}. \quad (5)$$

### A. Kinetic energy spectra

In order to go in detail in the results from all above ratios, let us understand the kinetic energy spectra of protons and neutrons for all type of fragments in the center of mass frame. The spectra of free protons and neutrons are very important experimental observables that can provide useful information about the particle production mechanism and reaction dynamics.

Fig. 1 displays the kinetic energy spectra for protons and neutrons at incident energy  $E = 50$  MeV/nucleon for semi-central geometry, while, Fig. 2 is at semi-peripheral geometry. The results are for neutron-rich  $^{132}\text{Sn} + ^{132}\text{Sn}$  and neutron-weak system  $^{112}\text{Sn} + ^{112}\text{Sn}$  by using the soft and stiff symmetry energy, respectively. The left and right panels in both figures are with the soft and stiff symmetry energy, while top, middle and bottom panels are for the free nucleons, and bound nucleons inside LCP's and IMF's, respectively.

It is clear from the figure that the production of neutrons is more favorable for neutron-rich system [34]. It is also true for all types of fragments as well as for the soft and stiff symmetry energy. This is due to the reason that in more neutron-rich system, the symmetry energy is more repulsive (attractive) for neutrons (protons) and hence more neutrons can be produced. On the second hand, the difference between yield or content of neutrons and protons decreases with the increasing of the kinetic energy. This is due to the Coulomb repulsion, which shifts the protons from low to high kinetic energy. The behavior is the same for all types of fragments. Interestingly, more neutrons can be produced with the soft symmetry energy for free particles as compared to the stiff one. The opposite is true for the LCP's and IMF's up to a certain kinetic energy and after that the same trend is observed just like for the free particles. It is an interesting phenomenon and unfortunately no one has noticed this one. This is due to the reason that the Coulomb effects are stronger **inside** LCP's and IMF's as compared to free particles. However, at a sufficient high kinetic energy, symmetry energy dominates over the Coulomb interactions and the behavior becomes just similar to that of free nucleons. Although, this intersection between soft and stiff symmetry energy for the fragments is not so clearly observed from here, so, we have extended the study with single ratio  $R_{N/Z}$  in the Fig. 3.

Let us move to the Fig. 2, which is displayed at semi-peripheral geometry. Almost, the same spectra are observed at semi-peripheral geometry, except for some exceptions. Once again, interesting point is that yield of

free neutrons at high kinetic energy for semi-central geometry (Fig. 1) is higher in comparison with the semi-peripheral one (Fig. 2). As we already knew that the symmetry energy/potential has two important functions: firstly, it tends to unbound more neutrons and secondly, it makes the neutrons more energetic than protons. Due to this, most of the finally observed neutrons are unbounded in the very early stage of the reaction as a result of the nucleon-nucleon (NN) collisions at semi-central collisions. Now, symmetry energy at pre-equilibrium time is just shifting the more neutrons towards the high kinetic energy. On the other hand, at semi-peripheral geometry, the emission of neutrons also depend on the symmetry potential/energy due to the relative lack of the NN collisions. The symmetry energy makes the neutrons unbound, but at relatively low kinetic energy. That is why, isospin effects are more pronounced at low kinetic energy for peripheral collisions and at high kinetic energy for central collisions. These results are also consistent with those in Ref. [6]. When one moves from LCP's to IMF's, the content of neutrons for neutron-rich system is lower than the neutron-weak system at high kinetic energies for semi-central as well as semi-peripheral geometries. This is due to the reason that at high kinetic energy the yield of free nucleons is higher as compared to fragments and it will result in more production of free neutrons for neutron-rich system as compared to the fragments.

### B. Single ratio

In order to make sure about the above discussion from Fig. 1 and Fig. 2, it is interesting to investigate the single ratio ( $R_{N/Z}$ ) of neutrons to protons for free nucleons, LCP's and IMF's, which is shown in Fig. 3 for neutron-rich system  $^{132}\text{Sn} + ^{132}\text{Sn}$  and neutron-weak system  $^{112}\text{Sn} + ^{112}\text{Sn}$  by using the soft and stiff symmetry energy. The left and right panels are at semi-central and semi-peripheral geometries, respectively. As is expected from Fig. 1 and Fig. 2, Fig. 3 depicts the results as follow:

- The isospin effects for more neutron-rich system are stronger and it is consistent with Ref. [6] and with Figs. 1, 2.
- $R_{N/Z}$  decreases with the kinetic energy for all types of fragments at semi-central as well as semi-peripheral geometries.
- For free nucleons, the isospin effects are stronger at high kinetic energy for semi-central geometry, while, the same is true at low kinetic energy for semi-peripheral geometries. It is also explained earlier in Ref. [6].
- The increase in the neutrons to protons ratio for neutron-rich system at sufficient high kinetic energy is due to the repulsive nature of the symmetry energy for neutrons.

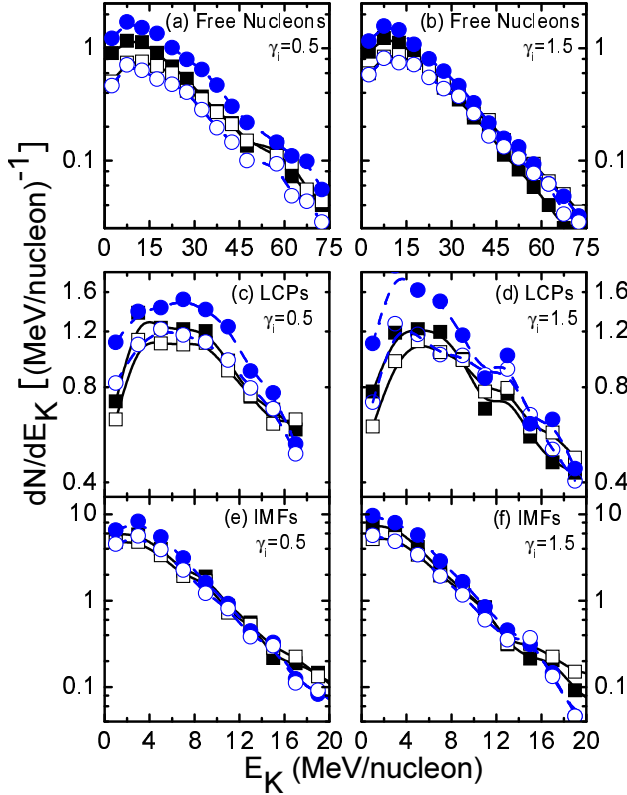


FIG. 1: (Color online) The kinetic energy spectra in the center of mass system for neutrons (solid symbols) and protons (open symbols) from the free nucleons (a,b), LCP's (c,d), and IMF's (e,f) at semi-central geometry ( $b = 2$  fm) of  $^{132}\text{Sn} + ^{132}\text{Sn}$  (blue circles) and  $^{112}\text{Sn} + ^{112}\text{Sn}$  (black squares) collisions at  $E = 50$  MeV/nucleon by using the soft (left) and stiff (right) symmetry energy, respectively.

- Let us discuss the single ratio for fragments.  $R_{N/Z}$  of IMF's is earlier studied by the Catania group using the BNV [18] and Texas group using the IBUU04 model [6]. Both models have different approaches for the symmetry energy and hence the results are little different from each other. In the BNV results, ratio decreases at low fragment kinetic energy and then increases at high kinetic energy for neutron-rich system with the stiff symmetry energy. On the other hand, in the IBUU04 calculations, the ratio is found to decrease with fragment kinetic energy. However, both the calculations have the same behavior with the soft and stiff symmetry energy. But, both groups have limited their study only to the relative low kinetic energy and were not able to investigate the cross-over phenomenon of symmetry energy, which takes place at higher kinetic energies and discussed in detail in

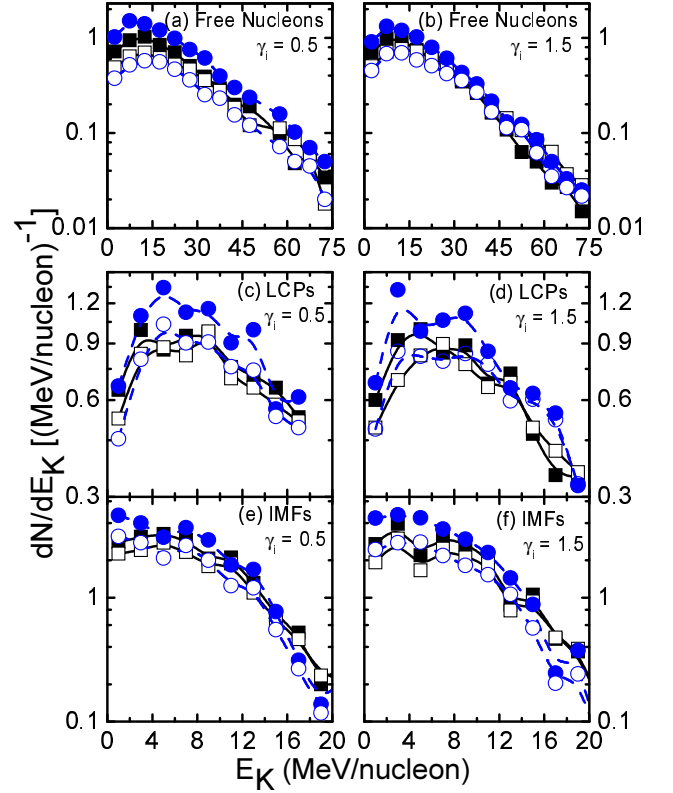


FIG. 2: (Color online) Same as in Fig. 1, but for the semi-peripheral geometry ( $b = 6$  fm).

this study.

- The large isospin effects are observed with the soft symmetry energy for free nucleons along whole range of the kinetic energy [20], while cross-over happens for the LCP's and IMF's at certain kinetic energy. Below the cross-over kinetic energy, the stiff symmetry energy produces larger neutrons to protons ratio and after the cross-over, it is true with the soft symmetry energy and behaves like just for free nucleons. Recently, Harmann *et al.* [35] displayed the data for single ratio from the IMF's below the cross-over kinetic energy. This data (not shown here) is favoring the soft symmetry energy in our studies with the IQMD, however, with the BNV, there data is well explained by the stiff symmetry energy. If one sees them carefully, we can find that the soft symmetry energy is more soft and stiff symmetry energy is less stiff in the BNV as compared to the IQMD and ImQMD models. In other words, the stiff symmetry energy from the BNV and the soft symmetry energy from the IQMD/ImQMD lies between the stiff symmetry en-



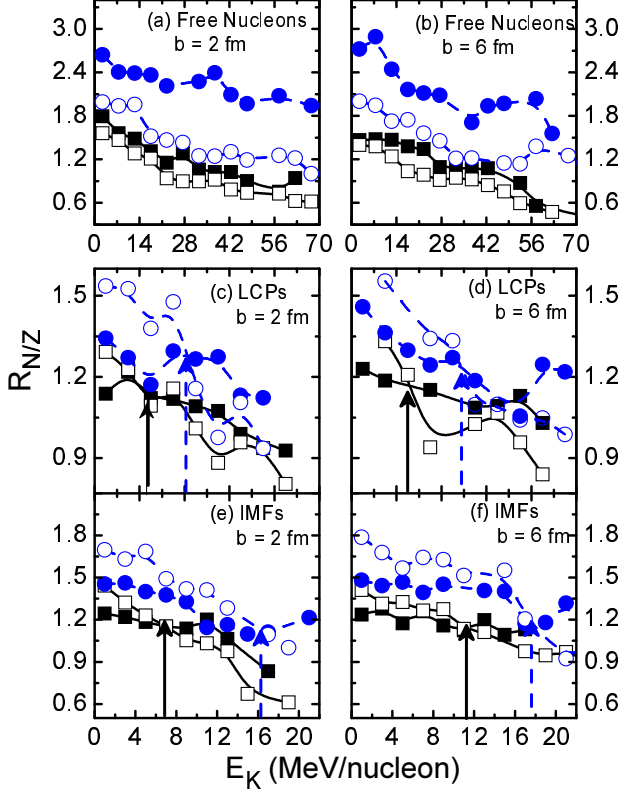


FIG. 3: (Color online) The ratio of neutrons to protons (a,b) at freeze-out time from free nucleons, LCP's (c,d) and IMF's (e,f) as a function of kinetic energy at semi-central (left) and semi-peripheral (right) geometries by using the soft (solid symbols) and stiff symmetry energy (open symbols). The vertical lines in the plots of LCP's and IMF's represent the kinetic energy at the cross-over points of the soft and stiff symmetry energy. Blue circle represents for  $^{132}\text{Sn} + ^{132}\text{Sn}$  and black squares for  $^{112}\text{Sn} + ^{112}\text{Sn}$  at  $E = 50$  MeV/nucleon.

ergy from the IQMD/ImQMD and the soft symmetry energy from the BNV calculations. It means that the data is favoring almost the same symmetry energy from both the models.

- The cross-over kinetic energy is at higher value for more neutron-rich system and increases with the size of the fragments (i.e. from LCP's to IMF's).
- The cross-over value of the kinetic energy also raises when one moves from semi-central to semi-peripheral geometries. This value is more affected for the more neutron-rich system.
- It is also clear from here that gas phase (free nucleons) is significantly enriched in neutrons relative to the liquid phase or fragments that are represented by the bounding nuclei. The phenomenon is known as isospin distillation/fractionation and is discussed many times in the literature only in term of the free

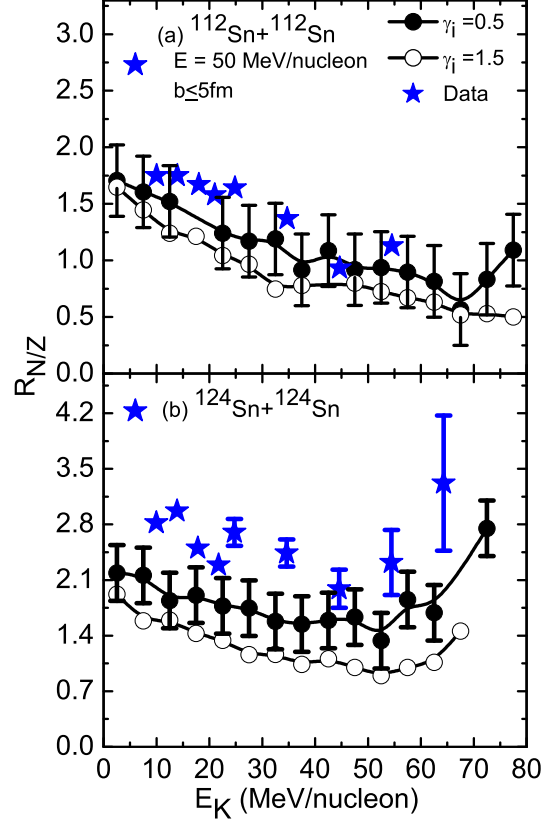


FIG. 4: (Color online) The comparison of neutrons to protons ratio from free nucleons, for the systems  $^{112}\text{Sn} + ^{112}\text{Sn}$  (top) and  $^{124}\text{Sn} + ^{124}\text{Sn}$  (bottom) at  $E = 50$  MeV/nucleon and impact parameter  $b \leq 5$ , with the experimental data of MSU/NSCL collaborations [13]. The filled and open circles represent the soft and stiff symmetry energies, respectively.

and bound nucleons [6]. More interesting results are expected for isospin distillation if one tries to study in term of different kind of fragments.

The theoretical results become more interesting and useful if one compares the results with experimental data. In the Fig. 4, we have, for the first time, compared the results of single ratio of neutrons to protons from free nucleons for neutron-weak system  $^{112}\text{Sn} + ^{112}\text{Sn}$  (in top panel) and for neutron-rich system  $^{124}\text{Sn} + ^{124}\text{Sn}$  (in bottom panel) at 50 MeV/nucleon with the experimental data [13]. The behavior of single ratio results for free nucleons is explained in the Fig. 3. The conclusion from the figure is that (1) more  $R_{N/Z}$  is observed for more neutron-rich system, which is also predicted by theoretical predictions, and (2)  $R_{N/Z}$  shows increment at higher kinetic energy, especially for  $^{124}\text{Sn} + ^{124}\text{Sn}$ . It indicates that the theoretical results are consistent with the experimental one.

The results are in good agreement with the soft symmetry energy except at very low and very high kinetic energy. The difference between soft and stiff symme-

try energy results for the neutron-weak system is almost comparable to the error bar, while, for neutron-rich system, the difference has a great importance over the error bar. In other words, the error bar of the theoretical results with the soft symmetry energy covers the error bar of the experimental data for both the systems under consideration. The difference at high kinetic energy between theoretical and experimental results for neutron-rich system is due to the large uncertainty in the measurement of the  $R_{N/Z}$ . By using the single ratio observable, one can reach at a partial conclusion that the asymmetric nuclear matter favors the soft symmetry energy at sub-saturation densities, which is also consistent with the other findings in the literature [4, 15–17, 19, 20, 24].

As we have observed, the single ratio mixes the symmetry energy with Coulomb effects throughout the kinetic energy range. In order to minimize the Coulomb effects and systematical error, it is reasonable to study the double neutrons to protons ratio for the isotopes of the same element. This is also studied in the literatures with only two isotopes [15, 16, 19, 20]. No one has tried to investigate the effect of double ratio on a series of isotopes in the asymmetric nuclear matter so far.

### C. Double ratio

In the present study, we consider reactions between three isotopes of Sn and observe the relative effect of these isotopes on the double ratio and symmetry energy. The pairs are as follow:  $^{132}\text{Sn} + ^{132}\text{Sn}$  and  $^{124}\text{Sn} + ^{124}\text{Sn}$ ,  $^{124}\text{Sn} + ^{124}\text{Sn}$  and  $^{112}\text{Sn} + ^{112}\text{Sn}$ ,  $^{132}\text{Sn} + ^{132}\text{Sn}$  and  $^{112}\text{Sn} + ^{112}\text{Sn}$ . The three pairs having the difference of 8, 12 and 20 neutrons, respectively. The universal behavior for double ratio is observed with the kinetic energy, i.e. with the increasing of kinetic energy, the double ratio is found to increase for all the three sets of isotopes which we have plotted in the Fig. 5. The increase in the double ratio is due to the effect that now energetic nucleons are more affected by the symmetry potential, which are already suppressed by the Coulomb repulsion in the single ratio results. The effect of symmetry energy on the double ratio is just like the single ratio, i.e. larger value with the soft symmetry energy as compared to the stiff one. Moreover, double ratio goes on increasing with the increase of the neutron difference between the pairs discussed above, or in other words, the double ratio from free nucleons goes on increasing with the initial-state double ratio of the systems from three different pairs of isotopes of Sn.

This increase is due to the effect that the more the number of neutrons, the more repulsive the symmetry energy for them. The Coulomb effects are already cancelled by taking the double ratio. Hence, the results are just like that as expected. The double ratio is found to be weakly sensitive towards the collision geometry. However, a little increase is observed at semi-peripheral geometry compared to semi-central one at high kinetic energy.

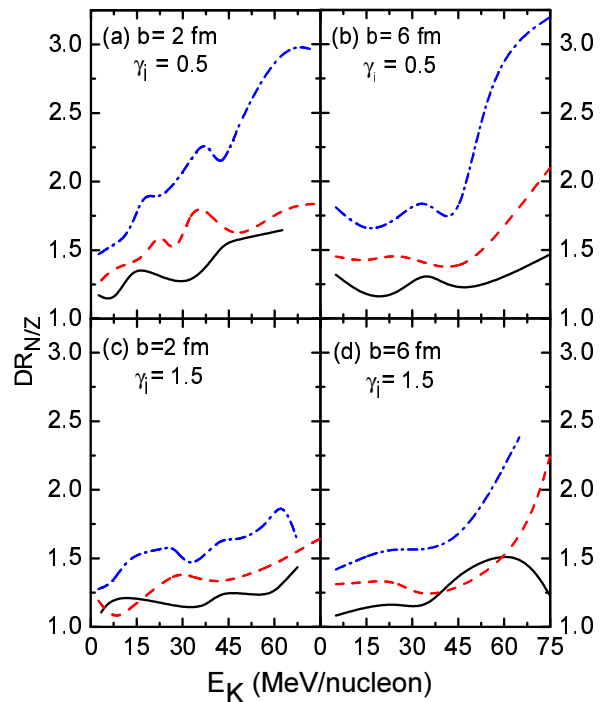


FIG. 5: (Color online) Free neutrons to protons double ratio, at semi-central (left) and semi-peripheral (right) geometry with the soft (top) and stiff (bottom) symmetry energy, as a function of kinetic energy at the incident energy  $E = 50$  MeV/nucleon. The different lines in the figure are the double ratio from different pairs: Solid line for  $^{132}\text{Sn} + ^{132}\text{Sn}$  and  $^{124}\text{Sn} + ^{124}\text{Sn}$ ; Dashed line for  $^{124}\text{Sn} + ^{124}\text{Sn}$  and  $^{112}\text{Sn} + ^{112}\text{Sn}$ ; Dash-dot line for  $^{132}\text{Sn} + ^{132}\text{Sn}$  and  $^{112}\text{Sn} + ^{112}\text{Sn}$ .

This is true with the stiff as well as the soft symmetry energy.

The double ratio has been studied many times in last couple of years by different groups with the help of the BUU97, IBUU04, BNV and ImQMD models and compared with the experimental results. Even so, we are still far away from the exact conclusion about the symmetry energy form. We have, along with all the possible results in the literature, compared the double ratio with the IQMD model in Fig. 6. Let us start with very first comparison of the BUU97 [15]. The results were very close to the experimental one, but, the reaction conditions were different. Firstly, in the BUU97 calculations, the incident energy was 40 MeV/nucleon, not 50 MeV/nucleon, just like the experimental one. Secondly, data set is only for the transverse emission, while in the BUU97 calculations, the nucleons used are emitted in all the directions. Move one step ahead to the IBUU04 results [16], where the symmetry energy is introduced with the help of momentum dependent interactions, the results are very far from the experimental data. The same is true for the BNV calculations performed by the Catania group in 2007 [17]. The most closeness between the data and the

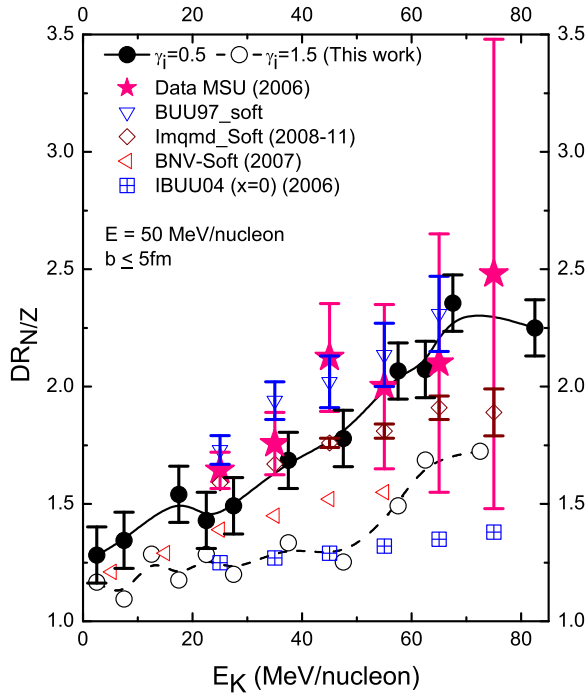


FIG. 6: (Color online) The comparison of free neutrons to protons double ratio at  $E = 50$  MeV/nucleon and  $b \leq 5$  with the MSU/NSCL data, BUU97, IBUU04, BNV, and ImQMD simulations.

calculation is observed by the ImQMD model in 2009 [19]. They found that the results with  $\gamma_i = 0.75$  are best fit with the experimental data for impact parameter  $b \leq 2$  fm. In the present study, we have performed simulations for  $b \leq 5$  fm and for the angular cuts, as mentioned in the experiments, with the soft and stiff symmetry energy and displayed the theoretical results over the whole range of the kinetic energy. The Fig. 6 clearly indicates that our results are very close to experimental data.

If we see the comparison of theoretical and experimental results from single (Fig. 4) and double ratio (Fig. 6) results, it seems that single ratio results require  $\gamma_i < 0.5$  to explain the data, and while the data is well explained by the  $\gamma_i = 0.5$  for double ratio. This is due to the reason that single ratio suffers the effect from the Coulomb interactions in addition to symmetry energy. As our main purpose is to extract the symmetry energy, where double ratio can act as a better candidate rather than the single ratio. In conclusion, the results of double ratio can be very well explained by the soft symmetry energy with  $\gamma_i = 0.5$  in comparison with single ratio, where the data is little underestimated by the theoretical predictions.

## IV. SUMMARY

In summary, we have performed a detailed analysis for kinetic energy spectra of free nucleons and bound nucleons inside fragments as well as the ratio parameters for the three reaction channels of Sn-isotopes at  $E = 50$  MeV/nucleon via multifragmentation. The kinetic energy spectra of protons and neutrons from free nucleons and all type of fragments in the center of mass frame shows that the content of neutrons is more favorable for neutron-rich system since the symmetry energy becomes more repulsive (attractive) for neutrons (protons) and hence more neutrons can be produced. In addition, the difference between yields or contents of neutrons and protons decreases with the increasing of the kinetic energy, which can be explained by the Coulomb repulsion shifts, making the protons from low to high kinetic energy. Interestingly, more free neutrons can be produced with the soft symmetry energy as compared to the stiff one.

From the single ratio of free neutrons to protons, it decreases with the kinetic energy for all types of fragments at semi-central as well as semi-peripheral geometries. However, the increase of the ratio from free nucleons for neutron-rich system is observed at sufficient high kinetic energy, this can be explained by the repulsive nature of the symmetry energy for neutrons. For single ratios of LCP's and IMF's, we noticed a transition at certain kinetic energy between the soft and stiff symmetry energy, while no transition for the free nucleons or gas phase. Below the cross-over kinetic energy, the stiff symmetry energy produces larger ratio of neutrons to protons and after the cross-over, it is true with the soft symmetry energy and behaves like just for free nucleons. This transition is also found to be strongly dependent on the isospin of the colliding partners, size of the fragment and weakly dependent on the collision geometry. Moreover, isospin distillation is also observed when one moves from the gas phase to liquid phase. It is further more interesting to study the isospin distillation in term of different kind of the fragments as compared to consider bound fragments as a single liquid phase.

The comparison of the theoretical results of single and double ratios with the experimental data emphasizes on the softness of the symmetry energy at sub-saturation densities, which is yet uncertain at the supra-saturation densities. Although, the single ratio study underestimates the data a little as compared to double ratio for the same stiffness of symmetry energy ( $\gamma_i = 0.5$ ), which reflects the double ratio is a relative good candidate for density dependence of symmetry energy at sub-saturation densities because of the cancel of Coulomb effect between two systems. Of course, the magnitude of double ratio of neutrons and protons from free nucleons strongly depends on the initial double ratio of the systems. It gives us an indication that it is better to study the isospin physics with a pair of  $^{132}\text{Sn} + ^{132}\text{Sn}$  and  $^{112}\text{Sn} + ^{112}\text{Sn}$ .

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